Title: Dynamic single-index scalar-on-function model

Abstract: Environmental exposures often exhibit temporal variability, such as air pollutions and weather conditions. There have been substantial interests in assessing how these time-dependent environmental exposures influence human health. While a significant portion of the current literature focused on modeling single time-dependent exposure, there is a burgeoning interest in studying the collective effects of multiple repeatedly measured exposures. In this paper, we bridge this gap by introducing a novel approach that combines multiple time-dependent exposures into a composite exposure mixture. We present the dynamic single-index scalar-on-function model, designed to explore the cumulative effect of this exposure mixture on health outcomes. The evolving relationship between the exposure mixture and health outcomes is characterized by a nonparametric bivariate function, named the "exposure-time-outcome intensity function." To estimate this bivariate function, we utilize B-spline tensor product bases and propose a profile algorithm for model estimation which facilitates to establish the large-sample properties for the resulting estimators. In addition, we introduce a nonparametric hypothesis testing procedure to determine if the effects of the exposure mixture vary over time. The performance of our proposed methods is examined through extensive simulation studies and is further illustrated using two real-world datasets.